CALCULABLE SPARTICLE MASSES WITH RADIATIVELY DRIVEN INVERTED MASS HIERARCHY

Howard Baer[1], Pedro Mercadante[1] and Xerxes Tata[2]

[1]*Department of Physics, Florida State University, Tallahassee, FL 32306 USA*

[2]*Department of Physics and Astronomy, University of Hawaii, Honolulu, HI 96822, USA*

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Abstract

Supersymmetric models with an inverted mass hierarchy (IMH: multi-TeV first and second generation matter scalars, and sub-TeV third generation scalars) can ameliorate problems arising from flavor changing neutral currents, CP violating phases and electric dipole moments, while at the same time satisfying conditions on naturalness. It has recently been shown that such an IMH can be generated radiatively, making use of infra-red fixed point properties of renormalization group equations given Yukawa coupling unification and suitable GUT scale boundary conditions on soft SUSY breaking masses. In these models, explicit spectra cannot be obtained due to problems implementing radiative electroweak symmetry breaking (REWSB). We show that use of $SO(10)$ D-term contributions to scalar masses can allow REWSB to occur, while maintaining much of the desired IMH. A somewhat improved IMH is obtained if splittings are applied only to Higgs scalar masses.

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Weak scale supersymmetry is especially appealing because destabilizing quadratic divergences that are present in the Standard Model cancel upon the introduction of supersymmetry [1]. Constraints from naturalness generally restrict superpartner masses to be below ~ 1 TeV, so that these new states of matter ought to be accessible to present or near future collider experiments [2]. In general, however, there also exist constraints on superpartner masses from flavor changing neutral current processes [3] (*e.g.* in the $K - \bar{K}$ system), electric dipole moments of the electron and neutron [4], $\mu \to e\gamma$ decays [5], proton decay [6] and Big Bang nucleosynthesis [7], that all favor superpartner masses in the multi-TeV range, unless specific assumptions such as universality [8] of scalar masses, or alignment [9] between fermionic and bosonic mixing matrices, are made. These considerations have provided a strong motivation to construct new and interesting mechanisms for the communication of supersymmetry breaking [10,11].

It is important to notice that naturalness arguments most directly apply to third generation superpartners, owing to their large Yukawa couplings. In contrast, the constraints from flavor physics mentioned above apply (mainly) to scalar masses of just the first two generations. This observation has motivated the construction of a variety of models, collectively known as inverted mass hierarchy (IMH) models [12], where the first and second generation squarks and sleptons have multi-TeV masses, while third generation scalars have sub-TeV masses. If the IMH already applies at or near the scale of grand unification, then it has been shown [13] that two-loop contributions to renormalization group (RG) running cause tachyonic third generation squark masses to occur, unless these masses are beyond ~ 1 TeV, which again pushes the model towards the "unnatural".

A resolution of this GUT scale dilemma has been presented recently in a series of papers [14,15] where the theme is that the IMH can be generated radiatively by starting with *all* scalar masses at the multi-TeV level at or near the GUT scale. It is pointed out that the infrared fixed point behaviour of the RG equations, together with a simple choice of boundary conditions, results in sub-TeV masses in the gaugino/Higgsino and third generation scalar sectors, while the first two generations of scalars are left in the multi-TeV range. An essential ingredient in the analysis is the presence of a singlet neutrino superfield $\hat{N}^c$ in addition to the usual superfields of the MSSM. The right-handed neutrino is expected to decouple at intermediate scales $Q = 10^{11} - 10^{13}$ GeV, leading to eV scale masses for the tau neutrino via the see-saw mechanism as is suggested by recent atmospheric neutrino data [16]

The most refined set of GUT scale boundary conditions [15] stipulate that

$$4m_Q^2 = 4m_U^2 = 4m_D^2 = 4m_L^2 = 4m_E^2 = 4m_N^2 = 2m_{H_u}^2 = 2m_{H_d}^2 = A_0^2, \qquad (1)$$

consistent with minimal SUSY $SO(10)$ unification. With boundary values for these parameters in the multi-TeV range, large suppression factors are generated for third generation and Higgs scalars, while other matter scalars remain heavy. The authors note that the radiatively driven IMH model has a problem with generating an appropriate radiative electroweak symmetry breaking (REWSB), which is common to all models at such high values of $\tan\beta \sim 50$, where Yukawa couplings most nearly unify. In some of the examples presented in Ref. [14] and [15], the squared Higgs masses never reach the negative values required by REWSB, while in other examples, various matter scalar squared masses are driven negative. Without an explicit mechanism for electroweak symmetry breaking, it is not possible to obtain mass spectra and couplings within this new and interesting picture.

In a recent paper [17], it has been shown that explicit mass spectra can be calculated in Yukawa unified $SO(10)$ models consistent with REWSB if D-term contributions to scalar masses are included at $Q = M_{GUT}$. The D-term contributions are expected whenever spontaneous symmetry breaking reduces the rank of the gauge group as *e.g.* when $SO(10)$ breaks to $SU(3) \times SU(2) \times U(1)$. Solutions with Yukawa coupling unification good to 5% were found only for positive D terms, which caused a split $m_{H_d} > m_{H_u}$ at the GUT scale, and only for negative values of the superpotential μ parameter.

In this paper, we examine the analogous solution to the problem of REWSB in the IMH model. We note that the analytic derivation of the GUT scale boundary condition discussed above used [15] one-loop RGEs applied only to soft SUSY breaking mass parameters that acquire multi-TeV scale masses. Terms in the RGEs proportional to sub-TeV quantities such as gaugino mass parameters were argued to be small, and dropped. In a realistic calculation, weak scale parameters and two-loop contributions to RGEs (if used) will modify the IMH solution somewhat. The expectation is that the desired qualitative features of an IMH will survive these additional perturbations. Our hope here is that there is some range of parameters for which a limited non-universality in scalar masses (originating in the D-terms) will allow REWSB to occur, while not spoiling too much of the expected IMH.

In our initial set of calculations, we assume that an $SO(10)$ SUSY GUT breaks to the MSSM plus a right-handed neutrino (MSSM+RHN) at a scale $Q = M_{GUT}$. At this scale, the scalar squared masses are given by

$$m_Q^2 = m_E^2 = m_U^2 = m_{16}^2 + M_D^2$$
$$m_D^2 = m_L^2 = m_{16}^2 - 3M_D^2$$
$$m_N^2 = m_{16}^2 + 5M_D^2$$
$$m_{H_{u,d}}^2 = m_{10}^2 \mp 2M_D^2,$$

where M_D^2 parametrizes the magnitude of the D-terms, and can, owing to our ignorance of the gauge symmetry breaking mechanism, be taken as a free parameter, with either positive or negative values. Here, m_{16} denotes the common mass of the 16-component spinor representation of $SO(10)$ to which the matter scalars including the right sneutrino belong, while m_{10} denotes the mass of the 10-component representation that contains the two Higgs doublets of the MSSM. The model is completely specified by the parameter set,

$$m_{16}, \ M_D^2, \ m_{1/2}, \ M_N, \ \tan\beta, \ sign(\mu),$$

where m_{10} and A_0 are determined in terms of m_{16} by the boundary condition above.

To calculate the superparticle and Higgs boson mass spectra, we adopt the bottom-up approach inherent in ISASUGRA, which is a part of the ISAJET program [18]. Our procedure is as follows. We generate random samples of model parameters

$$1000 < m_{16} < 10000 \ \text{GeV},$$
$$0 < m_{1/2} < 1000 \ \text{GeV},$$
$$0 < M_D^2 < m_{16}^2/3,$$
$$10 < \tan\beta < 55,$$
$$\mu > 0 \ \text{or} \ \mu < 0,$$

while allowing M_N to float between 5×10^{12} and 5×10^{13} GeV.

Starting with the three gauge couplings and t, b and τ Yukawa couplings of the MSSM at scale $Q = M_Z$ (or m_t), ISASUGRA evolves the various couplings up in energy until the scale where $g_1 = g_2$, which is identified as M_{GUT}, is reached. The GUT scale boundary conditions are imposed, and the full set of RGEs for gauge couplings, Yukawa couplings and relevant scalar masses are evolved down to $Q \sim M_{weak}$, where the renormalization group improved one-loop effective potential is minimized at an optimized scale choice $Q = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$ and radiative electroweak symmetry breaking is imposed. Using the new spectrum, the full set of SSB masses and couplings are evolved back up to M_{GUT} including weak scale sparticle threshold corrections to gauge and Yukawa couplings. The process is repeated iteratively until a stable solution is obtained. We use one loop RGEs for the soft SUSY breaking parameters, but two-loop equations for the gauge and Yukawa couplings.

Our first results are shown in Fig. 1. Here, we plot solutions to the superparticle mass spectrum consistent with REWSB, for $\mu < 0$ and $A_0 < 0$. The quantity S is the "crunch" factor defined as,

$$S = \frac{3(m_{u_L}^2 + m_{d_L}^2 + m_{u_R}^2 + m_{d_R}^2) + m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2 + m_{\tilde{\nu}_e}^2}{3(m_{\tilde{t}_1}^2 + m_{\tilde{b}_1}^2 + m_{\tilde{t}_2}^2 + m_{\tilde{b}_2}^2) + m_{\tilde{\tau}_1}^2 + m_{\tilde{\tau}_2}^2 + m_{\tilde{\nu}_\tau}^2}.$$

Notice that this differs slightly from the corresponding definition in Ref. [15] since we are able to use mass eigenvalues in the definition. In frame a), we show S versus the ratio M_D/m_{16}. We see that all solutions have *some* suppression of third generation masses, due to the large Yukawa couplings of the third generation. However, for $M_D/m_{16} \sim 0.2$, the value of S can reach values as high as 6-7. Indeed we see that most solutions have a significantly smaller value of S. This is in part due to our non-requirement of Yukawa coupling unification, which was assumed in the derivation of (1). In frame b), we show S versus $\tan\beta$. Here, it is easy to see that a maximum IMH develops for very large values of $\tan\beta$ where Yukawa unification can occur. The remaining frames show S versus a ratio indicating the degree of Yukawa coupling unification $R_{tb} = |(f_t - f_b)/f_t|$, where f_t, f_b and f_τ are the third generation Yukawa couplings evaluated at $Q = M_{GUT}$. $R_{\tau b}$ is similarly defined. In frames c) and d), we see that the maximum IMH is indeed obtained typically for the smaller values of R, where Yukawa couplings are most nearly unified. Similar results and suppression factors are obtained for $\mu > 0$ solutions, although in this case Yukawa coupling do not unify as well as for $\mu < 0$.

Two examples of specific spectra with considerable S factors and full $SO(10)$ D-terms are shown as case 1 and case 2 in Table 1. Case 1 has $\mu < 0$ and case 2 has $\mu > 0$. In case 1, first generation scalar masses are ~ 1500 GeV, while the lightest third generation squarks are $m_{\tilde{b}_1} = 310.9$ GeV and $m_{\tilde{t}_1} = 364.7$ GeV. The $\tilde{b}_1$ is the lightest third generation scalar, and is just beyond the region accessible to searches at the Fermilab Tevatron [19]. The $\widetilde{W}_1 \rightarrow \widetilde{Z}_1 W$ at $\sim 100\%$, while $\widetilde{Z}_2 \rightarrow \widetilde{Z}_1 Z^0$ or $\widetilde{Z}_1 h$ dominantly. In case 2, first generation scalars again have $m \sim 1500$ GeV, but in this case the top squark is the lightest third generation scalar ($m_{\tilde{t}_1} = 219.3$ GeV). The $\widetilde{W}_1$, with $m_{\widetilde{W}_1} = 124.1$ GeV, should be accessible to Fermilab Tevatron searches [20,21], since the $\widetilde{W}_1$ and $\widetilde{Z}_2$ decay via three-body modes which are dominated by W and Z exchange diagrams, so leptonic branching fractions are not suppressed.

Applying the $SO(10)$ D-terms to scalar masses upsets the precise form of the boundary condition of Eq. 1, and the values of S we obtain fall short of what has been obtained in

Ref. [15]. We also examined whether higher S values can be achieved by applying splitting only to the Higgs squared masses (since it is this splitting which allows for REWSB), leaving the matter scalars degenerate at M_{GUT}. While inconsistent with the $SO(10)$ framework, it adheres more closely to the boundary conditions in Eq. 1. We continue to parameterize the mass splitting in terms of the parameter M_D. These results may be relevant for scenarios with smaller GUT groups, but with a singlet neutrino. Our results with only splitting in the Higgs masses are presented in Fig. 2, again for $\mu < 0$ and $A_0 < 0$. We see in frame a) that in this case, somewhat larger S values up to $8 - 9$ can be obtained, but typically for values of $M_D/m_{16} \sim 0.4 - 0.6$. In frame b), we see that the crunch factor is again maximal for the largest values of $\tan\beta$ where Yukawa couplings most nearly unify. In frames c) and d), the higher S values are again obtained for the smallest values of R_{tb} and $R_{\tau b}$, $i.e$ where Yukawa coupling unification most nearly occurs.

In Table 1, we show two more cases (labelled 3 and 4) for mass splitting only in the Higgs sector. In case 3, first generation scalars have masses ~ 3000 GeV, while $m_{\tilde{t}_1} = 589.7$ GeV and $m_{\tilde{b}_1} = 581.5$ GeV. In this case, only the light Higgs scalar should be accessible to Fermilab Tevatron searches, while many sparticles should give observable signals at the CERN LHC pp collider, operating at $\sqrt{s} = 14$ TeV. In this case, $\widetilde{W}_1$ and $\widetilde{Z}_2$ decay with unsuppressed branching fractions into three-body modes, so that SUSY events at the LHC should be rich in isolated leptons, as well as b-jets from third generation squarks produced directly or as decay products of gluinos. In case 4, the first generation scalars have mass ~ 3300 GeV, while the lightest third generation scalar is $\tilde{t}_1$ with $m_{\tilde{t}_1} = 607$ GeV. The SUSY particles should again be beyond the reach of Fermilab Tevatron experiments, but should be accessible to LHC. The experimental signatures should again be rich in b-jets and isolated leptons produced in gluino and squark cascade decay events.

We have illustrated that the incorporation of D-terms allows the construction of models with REWSB where the first two generations of matter scalars have masses $\sim 2-3$ TeV, while other sparticle masses are in the sub-TeV range. Such a mass spectrum ameliorates (but does not completely cure) the flavor problem associated with SUSY models. The hierarchy that we obtain is significantly smaller that in the pioneering papers [14,15] where the requirement of REWSB was not implemented. In Ref. [15], the largest crunch factors are obtained for large values (> 1) of the unified Yukawa coupling and for relatively small values ($10^4 - 10^8$ GeV) of the right handed neutrino mass. In our study, the Yukawa coupling is typically smaller, and (motivated by the neutrino oscillation interpretation of the atmospheric neutrino data) we have fixed M_N to be $\sim 10^{13}$ GeV. Various refinements to get larger values of S together with REWSB are under investigation, as are the phenomenological consequences of IMH models.

REFERENCES

[1] For recent reviews, see *e.g.* S. Martin, in *Perspectives on Supersymmetry*, edited by G. Kane (World Scientific), hep-ph/9709356; M. Drees, hep-ph/9611409 (1996); J. Bagger, hep-ph/9604232 (1996).

[2] R. Barbieri and G. Giudice, Nucl. Phys. **B306**, 63 (1988); G. Anderson and D. Castano, Phys. Lett. **B347**, 300 (1995) and Phys. Rev. **D52**, 1693 (1995).

[3] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. **B477**, 321 (1996).

[4] W. Fischler, S. Paban and S. Thomas, Phys. Lett. **B289**, 373 (1992); S. Dimopoulos and L. Hall, Phys. Lett. **B344**, 185 (1995); T. Ibrahim and P. Nath, Phys. Rev. D**57**, 478 (1998); *ibid.*, erratum, Phys. Rev. D**58**, 019901 (1998).

[5] L. Hall, V. Kostelecky and S. Raby, Nucl. Phys. **B267**, 415 (1986); R. Barbieri and L. Hall, Phys. Lett. **B338**, 212 (1994); J. Hisano, D. Nomura, Y. Okada, Y. Shimizu, M. Tanaka, Phys. Rev. D**58**, 116010 (1998).

[6] T. Goto and T. Nihei, Phys. Rev. **D59**, 115009 (1999), and references therein.

[7] G. Coughlan, W. Fischler, E. Kolb, S. Raby and G. Ross, Phys. Lett. **B131**, 59 (1983); J. Ellis, D. Nanopoulos and M. Quiros, Phys. Lett. **B174**, 176 (1986); T. Moroi, M. Yamaguchi and T. Yanagida, Phys. Lett. **B342**, 105 (1995); M. Kawasaki, T. Moroi and T. Yanagida, Phys. Lett. **B370**, 52 (1996).

[8] S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981).

[9] Y. Nir and N. Seiberg, Phys. Lett. **B309**, 337 (1993).

[10] M. Dine, A. Nelson, Y. Nir and Y. Shirman, Phys. Rev. **D53**, 2658 (1996).

[11] L. Randall and R. Sundrum, hep-th/9810155 (1998); T. Gherghetta, G. Giudice and J. Wells, hep-ph/9904378 (1999).

[12] M. Drees, Phys. Rev. D**33**, 1468 (1986); S. Dimopoulos and G. Giudice, Phys. Lett. **B357**, 573 (1995); A. Pomarol and D. Tommasini, Nucl. Phys. **B466**, 3 (1996); A.G. Cohen, D.B. Kaplan, and A.E. Nelson, *Phys. Lett. B***388**, 588 (1996).

[13] N. Arkani-Hamed and H. Murayama, *Phys. Rev. D***56**, R6733 (1997); K. Agashe and M. Graesser, *Phys. Rev. D***59**, 015007 (1999).

[14] J. Feng, C. Kolda and N. Polonsky, *Nucl. Phys. B* **546**, 3 (1999); J. Bagger, J. Feng and N. Polonsky, hep-ph/9905292 (1999).

[15] J. Bagger, J. Feng, N. Polonsky and R. Zhang, hep-ph/9911255 (1999).

[16] Y. Fukuda *et al. Phys. Rev. Lett.* **82**, 2644 (1999).

[17] H. Baer, M. Diaz, J. Ferrandis and X. Tata, hep-ph/9907211 (1999); H. Baer. M. Brhlik, M. Diaz, J. Ferrandis, P. Mercadante, P. Quintana and X. Tata, in preparation.

[18] F. Paige, S. Protopopescu, H. Baer and X. Tata, hep-ph/9804321 (1998).

[19] H. Baer. P. Mercadante and X. Tata, Phys. Rev. D**59**, 015010 (1999). That inclusion of D-terms could facilitate radiative electroweak symmetry breaking had been previously pointed out by, R. Rattazzi and U. Sarid, *Phys. Rev. D* **53**, 1553 (1996) and H. Murayama, M. Olechowski and S. Pokorski, *Phys. Lett. B* **371**, 57 (1996).

[20] Some recent papers include V. Barger and C. Kao, Phys. Rev. **D60**, 115015 (1999); K. Matchev and D. Pierce, Phys. Rev. **D60**, 075004 (1999) and Phys. Lett. **B467**, 225 (1999); H. Baer, M. Drees, F. Paige, P. Quintana and X. Tata, hep-ph/9906233 (1999).

[21] Some phenomenological aspects of IMH models have been examined by V. Barger, C. Kao and R-J. Zhang, hep-ph/9911510 (1999).

TABLES

TABLE I. Weak scale sparticle masses and parameters (GeV) for four IMH model case studies. The first two cases contain full $SO(10)$ D-terms applied to GUT scale SSB scalar masses. The last two cases have splittings applied only to the Higgs scalar masses.

parameter	case 1	case 2	case 3	case 4
m_{16}	1490.9	1363.9	2824.0	3239.9
m_{10}	2108.5	1928.8	3993.8	4581.9
M_D	371.7	276.9	1266.1	1503.7
$m_{1/2}$	380.3	338.3	570.9	473.5
M_N	8.03×10^{12}	2.04×10^{13}	3.63×10^{13}	1.42×10^{13}
A_0	-2981.8	-2727.8	-5648.0	-6479.7
$\tan \beta$	47.5	48.1	52.5	48.3
$m_{\tilde{g}}$	966.9	869.9	1417.3	1223.3
$m_{\tilde{u}_L}$	1716.0	1552.6	3036.3	3374.7
$m_{\tilde{d}_R}$	1530.8	1436.3	3022.6	3372.6
$m_{\tilde{\ell}_L}$	1365.9	1296.3	2825.2	3221.4
$m_{\tilde{\ell}_R}$	1553.0	1404.1	2884.4	3314.1
$m_{\tilde{\nu}_e}$	1363.6	1293.8	2824.1	3220.4
$m_{\tilde{t}_1}$	364.7	219.3	589.7	606.8
$m_{\tilde{t}_2}$	835.0	740.3	1111.3	1187.8
$m_{\tilde{b}_1}$	310.9	405.8	581.5	918.0
$m_{\tilde{b}_2}$	728.0	616.6	888.9	1027.2
$m_{\tilde{\tau}_1}$	777.8	515.4	632.2	949.0
$m_{\tilde{\tau}_2}$	825.3	732.8	1736.5	1989.8
$m_{\tilde{\nu}_\tau}$	780.4	728.0	1734.6	1988.1
$m_{\widetilde{W}_1}$	287.8	124.1	308.7	267.0
$m_{\widetilde{Z}_2}$	288.2	147.2	316.9	274.9
$m_{\widetilde{Z}_1}$	160.6	100.6	238.2	195.2
m_h	125.5	117.6	111.3	104.3
m_A	743.9	415.3	1244.0	2238.7
m_{H^+}	750.8	427.0	1248.5	2242.1
μ	-359.0	136.5	-321.2	285.7
R_{tb}	0.144	0.089	0.079	0.074
$R_{\tau b}$	0.064	0.147	0.094	0.163

FIG. 1. The crunch factor S versus $a)$ $\frac{M_D}{m_{16}}$ and $b)$ $\tan\beta$. In frames $c)$ and $d)$, we plot the Yukawa coupling unification ratios R_{tb} and $R_{\tau b}$ defined in the text. These models include $SO(10)$ D-terms, and have $\mu < 0$.
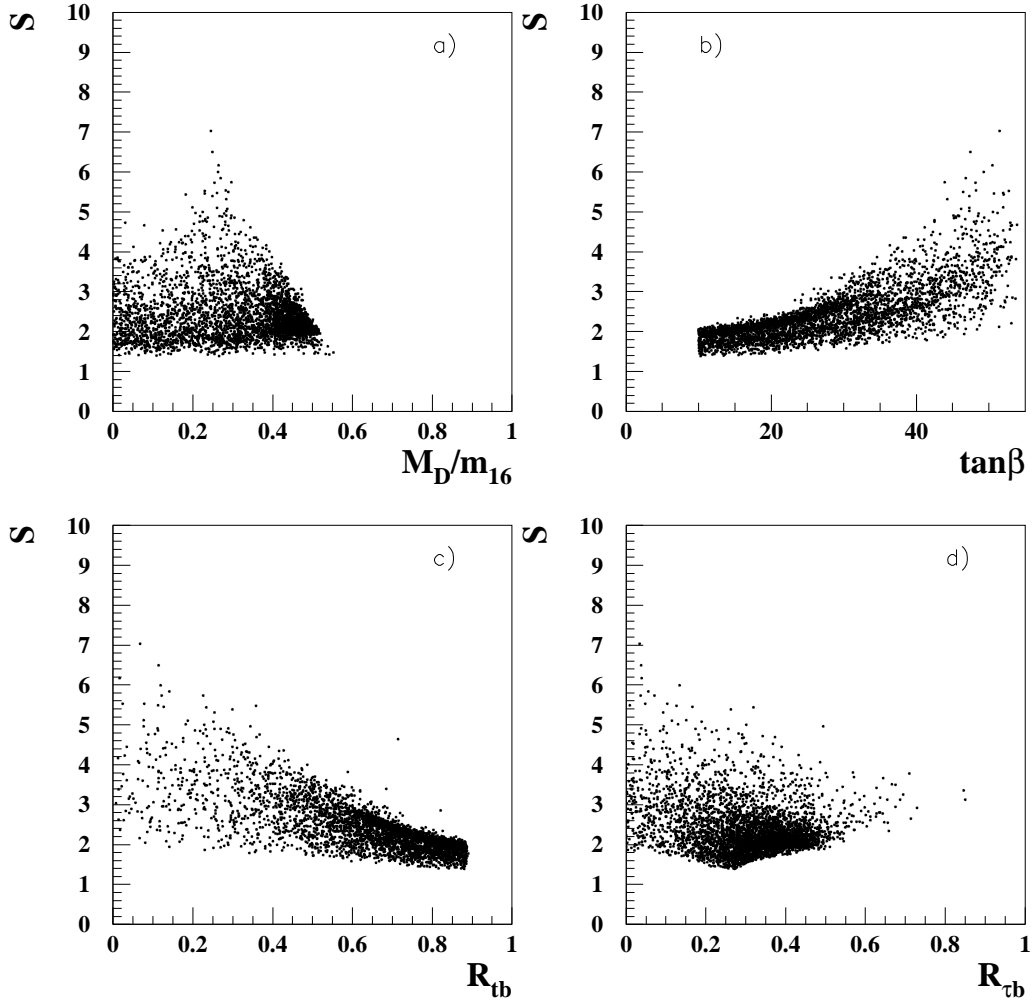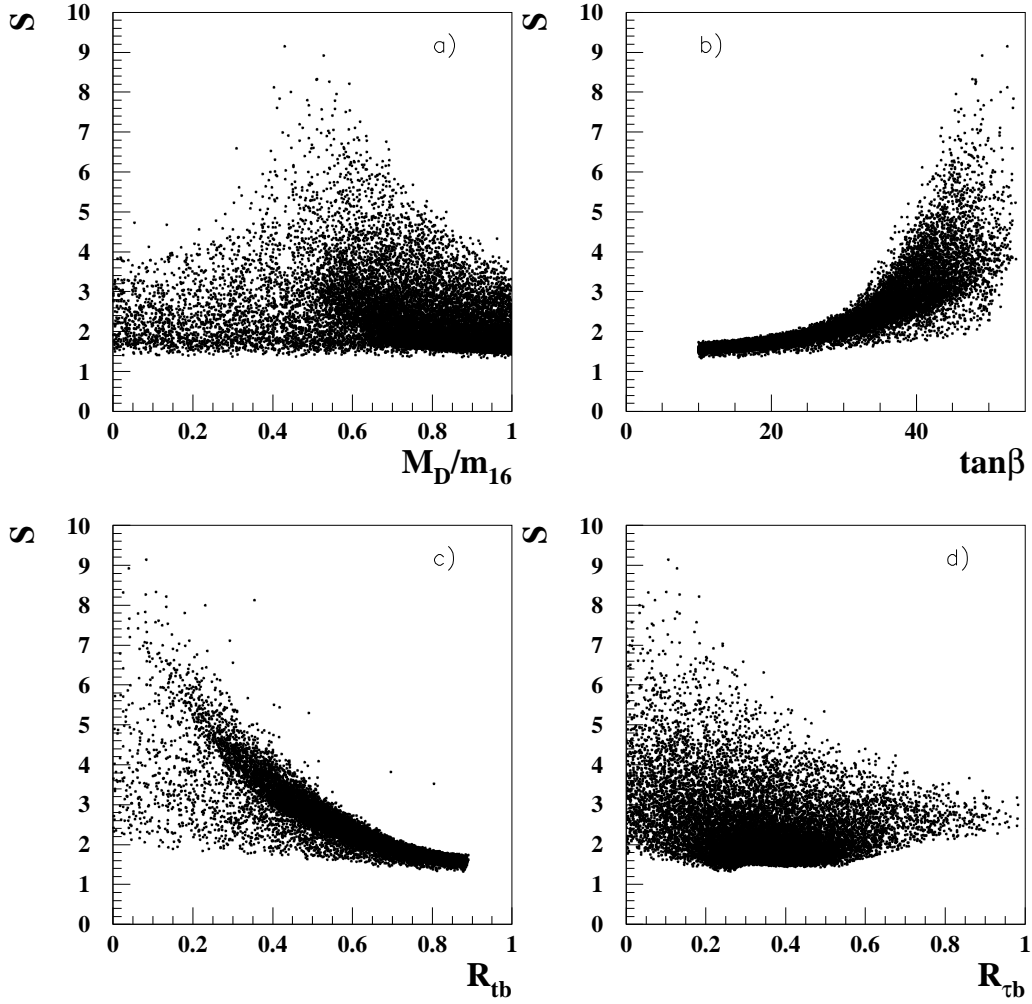
FIG. 2. The crunch factor S versus a) $\frac{M_D}{m_{16}}$ and b) $\tan\beta$. In frames c) and d), we plot the Yukawa coupling unification ratios R_{tb} and $R_{\tau b}$. These models apply splittings only to the Higgs scalars at M_{GUT}. We take $\mu < 0$.